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SESSION 7: SYSTEM AND RECEIVER NOISE PERFORMANCE CLINIC

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7.0 Elementary Considerations of Noise Performance

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It is generally recognized that the output noise of a receiving system contains components contributed not only by the generator at the input of the receiving system but also by the receiver itself. Furthermore, the evaluation of the output signal-to-noise ratio of the system will depend not only on the output noise but also on the nature of the signal that is impressed on the input of the receiver and that appears in the output utilization circuit. Hence, any meaningful evaluation of noise performance of receivers when they are used in a particular system must include considerations, under operating conditions, of the sources that contribute to the output noise, the bandwidth and gain of the receiving system in all of its responses, the nature of the signal and the efficacy of the output utilization circuit. It is evident that no single number can describe completely how well a given receiver will perform in all kinds of systems.

What, then, are the pertinent attributes which we are seeking, and how are they measured and quoted?

From the viewpoint of the designer of the receiver, the attributes must be readily measurable. From the viewpoint of the marketer, the quoted numbers should represent the best possible performance of the receiver when it is used to its utmost capability in a system designed to match that capability. The system's engineer must be assured that the quoted numbers will enable him to calculate the output signal-to-noise ratio. It is his responsibility to match his signal to the band width of the receiver and to know what penalty is paid when he doesn't. This represents no hardship, in general, for he can always introduce appropriate matching filters in his system that tend to optimize the output signal-to-noise ratio.

It is also the responsibility of the system's engineer to employ the best possible utilization circuit at the output of the receiver. The nature of the utilization circuit depends on the nature of his signal, whether it is AM, FM, single-sideband, double sideband, sky noise etc. These things are established by the system's engineer. They are very important, indeed, and must be considered carefully.

Let us assume that the system's engineer accepts the responsibility of matching the noise bandwidth to his signal bandwidth and of using the most efficient detection for his signal. What else must he know about his receiving system to predict its noise performance? He must know the gain-frequency characteristics of the receiver. Does it have only one response or multiple responses? He must know the environment in which he plans to use the receiver. What is the antenna temperature, or generator temperature? He must know how much of his output noise originates within the receiver itself. What is the effective input noise temperature of the receiver? How does it compare with his antenna temperature?

The antenna temperature and the effective input noise temperature of the receiving system can be used to compute an operating temperature, T_{op}, for the receiving system. The operating temperature appears to be a simple number for the system's engineer to use in evaluating his system noise performance. The expression ($k\ T_{op}\ B_o$) represents the power required of an input signal to make the output signal-to-noise ratio unity.

The pertinent attributes that are measurable and quotable are: 1) The gain at each of the responses, 2) The bandwidth, and 3) The multiple response or "broad-band" effective input noise temperature.

The concepts of operating noise temperature and multiple response or "broad-band" effective input noise temperature require further discussion and definition. The relation between operating noise temperature, generator temperature, effective input noise temperature and noise factor will be brought out during the discussion.

OPERATING NOISE TEMPERATURE. The noise performance of any particular system is evaluated in terms of its output signal-to-noise power ratio under operating conditions, $\frac{S_o}{N_o}$. The output signal power can always be expressed as the signal power available at the input terminals multiplied by the signal gain, G_s.

Definition:

The transducer gain, G_s, for the signal is the ratio of (1) the total signal power delivered to the output utilization circuit, S_o, to (2) the total signal power available at the input terminals, S_i. (Note that G_s does not imply a one to one correspondence between input and output frequencies.)

The output noise power can be expressed in terms of the signal gain, G_s, the output signal bandwidth, B_o,* and an operating noise temperature, T_{op}, thus: (See Appendix III)

$$N_o = G_s\ k\ T_{op}\ B_o \tag{1}$$

Hence the output signal-to-noise ratio is:

$$\frac{S_o}{N_o} = \frac{G_s\ S_i}{G_s\ k\ T_{op}\ B_o} = \frac{S_i}{k\ T_{op}\ B_o} \tag{2}$$

From this it is clear that two receiving systems will exhibit the same output signal-to-noise ratio if they have the same $\frac{S_i}{T_{op}\ B_o}$ ratio.

For instance, two receiving systems having the same available input signal power per unit output bandwidth must have the same operating noise temperature to produce the same signal-to-noise ratio at the output.

Let us now apply the foregoing concepts first, to single response receivers and second, to multiple response receivers.

SINGLE RESPONSE RECEIVERS. In the single response** receiver, the output noise power may be written thus:

$$N_o = G_s \, k \, B_o \, T_g + N_N \tag{3}$$

where N_N is the output noise power which originates within the receiver

T_g is the input generator noise temperature.

The output noise power which originates within the receiver can be expressed in terms of a temperature, thus:

$$N_N = T_e \, G_s \, k \, B_o \tag{4}$$

so that:

$$N_o = G_s \, k \, B_o \, (T_g + T_e) \tag{5}$$

The term "T_e" is called "The Effective Input Noise Temperature"[1] (See Appendix I) and the sum of the input generator noise temperature and the effective input noise temperature is seen to be the operating noise temperature, T_{op}; thus:

$$T_{op} = T_g + T_e \tag{6}$$

To measure T_e, the designer can observe his output noise power for two different temperatures of the generator. If the ratio of the two output noise powers is "Y", we have from equation (5):

$$Y = \frac{T_g\,(\text{hot}) + T_e}{T_g\,(\text{cold}) + T_e} \tag{7}$$

from which:

$$T_e = \frac{T_g\,(\text{hot}) - Y \, T_g\,(\text{cold})}{Y - 1} \tag{8}$$

The concept of effective input noise temperature as used to evaluate the noise performance of a receiver is a simple one to understand. Prior to its introduction in the literature by Gordon and White[2] in 1958, another concept, that of Noise Factor (Noise Figure), had been used quite generally. This was introduced in the literature by D. O. North[3] in 1942 and by H. T. Friis[4] in 1944. The I. R. E. defined it first in 1952[5] and subsequently in 1957[6]. There are only minor differences between the definitions of North, Friis and the I. R. E. Other definitions of noise figure and noise factor have appeared elsewhere in the literature and have created some unnecessary and undesirable dilemmas. The panel advocates the acceptance and use of the I. R. E. definition of noise factor (noise figure) (see Appendix II).

From the compatible definitions of North, Friis and the I. R. E. it is clear that the noise factor (noise figure) can be expressed by the relation:

$$F = \frac{G_s \, k \, 290 \, B_o + N_N}{G_s \, k \, 290 \, B_o} \tag{9}$$

Solving for N_N we have

$$N_N = (F - 1) \, G_s \, k \; 290 \; B_o \tag{10}$$

Substituting this expression for N_N in equation (3) we have:

$$N_o = G_s \, k \, B_o \left\{ T_g + (F - 1) \; 290 \right\} \tag{11}$$

The relation between noise figure and effective input noise temperature may be seen by comparing equation (11) with equation (5) to obtain:

$$T_e = (F - 1) \; 290 \tag{12}$$

To evaluate the noise figure from the "Y" factor, as defined above, we use equation (11) to derive the expression:

$$F = \frac{\left(\frac{T_g\,(\text{hot})}{290} - 1\right) - Y\left(\frac{T_g\,(\text{cold})}{290} - 1\right)}{Y - 1} \tag{13}$$

Often the cold temperature is assumed to be 290°, and the approximate relation is obtained:

$$F \approx \frac{\frac{T_g\,(\text{hot})}{290} - 1}{Y - 1} \tag{14}$$

The concepts of effective input noise temperature, T_e, and noise factor, F, for single response receivers are equally acceptable by the component designer, the marketer and the system's engineer, provided they all agree to accept and use the recommended I. R. E. definitions. It appears that there is more general agreement on the definition of T_e than of F. T_e is generally easier to think about, especially when the receiver is a good one. For poorer receivers, it is sometimes more convenient to use noise factor expressed in decibels ($10 \log_{10} F$) than either T_e or F.

The operating noise temperature was shown in equation (6) to be

$$T_{op} = T_g + T_e$$

and, by using equation (12) may also be written in terms of noise figure, thus:

$$T_{op} = T_g + (F - 1) \; 290 \tag{15}$$

MULTIPLE RESPONSE† RECEIVERS. The foregoing brief description applies to the situation regarding single response receivers. For receiving systems that are capable of receiving signals and/or noise on more than one response, one may proceed along entirely analogous lines.

Examples of such systems include superheterodyne receivers that may have a response at the image frequency and sometimes at higher frequencies near the harmonics of the beating oscillator. Also, systems using parametric amplifiers may have a response at the idler frequency.

Again let us approach this case by evaluating the total output noise under operating conditions. We shall describe the output signal-to-noise ratio in the case when the signal may occupy either one or several of the input responses. It is clear that we must consider the following items:

(1) The contributions to the output noise power due to the noise power available from the impedance which is connected to the accessible input terminals under operating conditions. These conditions can be described by assigning appropriate noise temperatures to the generator impedance at all of the various responses.

(2) All other contributions to the output noise power. These are due to noise generated within the receiver components as well as noise resulting from any frequency conversions internal to the receiving system. (These are not introduced at the accessible input terminals of the system.)
(3) The total output signal power delivered to the output utilization circuit, taking due account of all responses of the system which contribute to the output signal.

If we denote the portion of the output noise power described by (1) above by N_{go} and that described by (2) above by N_N, we can write the total output noise power, N_o, thus:

$$N_o = N_{go} + N_N \tag{16}$$

Letting B_N be the limiting noise bandwidth common to all responses, we can write:

$$N_{go} = k B_N (T_{g1} G_1 + T_{g2} G_2 + \dots + T_{gn} G_n) \tag{17}$$

where G_n is the transducer gain of the n^{th} response. It is the ratio of (1) the output power delivered to the utilization circuit to (2) the corresponding input power available to the n^{th} input response. (In a system in which we cannot ascribe a common limiting bandwidth to all responses, the individual bandwidths must be carried throughout the computation of N_o).

In line with our earlier analysis, it is convenient to characterize N_N by a temperature, T_b, common to all responses.

$$N_N = k B_N T_b (G_1 + G_2 + \dots + G_n) \tag{18}$$

The total output noise is then:

$$N_o = k B_N \{G_1 (T_{g1} + T_b) + G_2(T_{g2} + T_b) + \dots + G_n (T_{gn} + T_b)\} \tag{19}$$

As before, in the single response case, we can characterize the noise performance of the receiving system in terms of an operating noise temperature, T_{op}, as defined by equation (1):

$$T_{op} = \frac{N_o}{k B_o G_s} \tag{20}$$

where B_o and G_s are as previously defined. (See Appendix III).

It is of fundamental importance to establish clearly how the temperature, T_b, which characterizes the noisiness of the receiver, can be determined by measurement. Since most modern noise generators used in noise measurements generate broadband noise (noise diodes, gas discharge lamps, hot and cold body loads) the direct measurement is one in which noise is injected <u>equally</u> into <u>all</u> responses. In other words, the measurement conditions are usually such that

$$T_g = T_{g1} = T_{g2} = \dots = T_{gn}$$

Hence, equation (19) becomes:

$$N_o = k B_N (T_g + T_b)(G_1 + G_2 + \dots + G_n). \tag{21}$$

To measure T_b, we observe the output noise power for two different temperatures of the noise generator, T_g (hot) and T_g (cold), to obtain:

$$\frac{N_o\text{ (hot)}}{N_o\text{ (cold)}} = Y = \frac{T_g\text{ (hot)} + T_b}{T_g\text{ (cold)} + T_b} \tag{22}$$

from which:

$$T_b = \frac{T_g\text{ (hot)} - Y\,T_g\text{ (cold)}}{Y - 1} \tag{23}$$

T_b may be called the multiple channel or "broad-band" effective input noise temperature of the receiver, since it was obtained by a measurement procedure identical to that used for obtaining T_e. (Equation (8) is used for the single response case.)

T_b is related to the multiple channel or "broad-band" noise figure, F_b. To evaluate F_b from the "Y" factor as defined above, one uses the equation:

$$F_b = \frac{\left(\frac{T_g\text{ (hot)}}{290} - 1\right) - Y\left(\frac{T_g\text{ (cold)}}{290} - 1\right)}{Y - 1} \tag{24}$$

This is exactly the same relation as was obtained for the noise figure, F, for the single response case (eq. 13). It is seen from equations (23) and (24) that the relation between the multiple channel noise figure and the multiple channel effective input noise temperature is:

$$T_b = (F_b - 1)\ 290 \tag{25}$$

(The multiple channel noise figure agrees with the I.R.E. definition of noise figure when the input signal is distributed equally in all input responses of the system, i.e., when all responses are considered to be "the principal frequency transformation.")

Since, in general, the input signal applied to multiple response receivers may or may not occupy all input responses, we must consider the two cases separately and in more detail, (A) when the input signal occupies only one response, and (B) when the input signal occupies more than one response of the receiving system.

(A) Signal Input at Only One Response

For the specific case when the input signal occupies only response number one, $G_s = G_1$, and from equations (19) and (20) we have:

$$T_{op} = \frac{B_N}{B_o}\left[T_{g1} + T_b + \frac{G_2}{G_1}(T_{g2} + T_b) + \frac{G_3}{G_1}(T_{g3} + T_b) + \dots + \frac{G_n}{G_1}(T_{gn} + T_b)\right] \tag{26}$$

The bandwidth ratio, $\frac{B_N}{B_o}$, is equal to or greater than unity; ($\frac{B_N}{B_o} \geq 1$). The lowest operating noise temperature obtains when the noise bandwidth B_N, matches the signal bandwidth, B_o. Now it is convenient to assume that the system's engineer will take care of this in his system, and then equation (26) becomes:

$$T_{op} = T_{g1} + T_b + \frac{G_2}{G_1}(T_{g2} + T_b) + \frac{G_3}{G_1}(T_{g3} + T_b) + \dots + \frac{G_n}{G_1}(T_{gn} + T_b) \tag{27}$$

T_b can be substituted from equation (23) into equation (26) or (27).

For the special case when, under operating conditions, the generator noise temperatures applied to all input responses are equal, equation (27) reduces to:

$$T_{op} = (T_g + T_b)(1 + \frac{G_2}{G_1} + \ldots\ldots + \frac{G_n}{G_1}) \tag{28}$$

(B) Signal Input at More Than One Response

If we now wish to evaluate T_{op} for the case where the received input signal is distributed over more than one input response, we note that only G_s can be affected in the equation

$$T_{op} = \frac{N_o}{k\ B_o G_s} \tag{20}$$

(N_o is, of course, unaffected in a linear system and we assume that B_o, the signal output bandwidth, remains unaffected also.)

When the portions of the input signal that are received by the various responses are totally uncorrelated, with their powers denoted by S_{i1}, S_{i2}, ... S_{in},

then

$$S_o = S_i\ (\text{total})\ G_s = S_{i1}\ G_1 + S_{i2}\ G_2 + \ldots\ S_{in}\ G_n \tag{29}$$

or:

$$G_s = \frac{S_{i1}\ G_1 + S_{i2}\ G_2 + \ldots\ S_{in}\ G_n}{S_{i1} + S_{i2} + \ldots\ S_{in}} = \frac{S_o}{S_i\ (\text{total})} \tag{30}$$

We again obtain T_{op} from equations (19) and (20) by substituting G_s:

$$T_{op} = \frac{N_o}{k\ B_o G_s} = \frac{B_N\ [G_1 (T_{g1} + T_b) \ldots + G_n (T_{gn} + T_b)]}{B_o \left(\frac{S_{i1}\ G_1 + S_{i2}\ G_2 + \ldots\ S_{in}\ G_n}{S_{i1} + S_{i2} + \ldots\ S_{in}} \right)} \tag{31}$$

For the simple case where $S_{i1} = S_{i2} = \ldots = S_{in}$, we obtain $G_s = (G_1 + G_2 + \ldots G_n)/n$, and with $B_N = B_o$,

$$T_{op} = \frac{G_1\ (T_{g1} + T_b) + \ldots\ G_n\ (T_{gn} + T_b)}{\frac{1}{n}\ (G_1 + G_2 + \ldots\ G_n)} \tag{32}$$

For the equally simple case where the response gains are equal, $G_1 = G_2 \ldots. = G_n$, (but arbitrary S_i's), we have $G_s = G_1$ and

$$T_{op} = (T_{g1} + T_b) + \ldots (T_{gn} + T_b). \tag{33}$$

T_b, of course, is obtained as before from equation (23) above.

From the above discussion it can be seen that when the received input signal is distributed over several responses incoherently, G_s is never larger than it would be if the received signal were entirely in the response exhibiting the largest gain. Hence, for the case where all response gains are equal, G_s and thereby T_{op} equation (33) are independent of how the (uncorrelated) input signal is distributed over the various input responses.

But with T_{op} (and B_o) constant, the output signal-to-noise power ratio

$$\frac{S_o}{N_o} = \frac{S_i}{k\ T_{op}\ B_o}$$

is seen to depend only on the total input signal power.

In receiving man-made signals, this total input signal power is fixed by the transmitter's capability. If we choose to spread it incoherently over several frequency bands - corresponding to the several responses of our receiving system - the power input to each response simply drops and nothing is gained.

However, if no limitation of transmitter power exists (as, for instance, in the broad-band radiometry case) the total input signal to our receiving system is proportional to the number of its input responses. This results in a corresponding improvement of $\frac{S_o}{N_o}$ over some other receiving system having the same T_{op} but only one input response. For instance, assuming equal gain and equal signal densities in all input responses, $\frac{S_o}{N_o}$ of an n - input response receiving system is equal to that obtained with a single response system having an operating noise temperature equal to 1/n times that of the n - response system.

Let us now briefly consider the case where the portions of the input signal that are received by the various responses are partially or totally correlated. For this case, the gain G_s will be a more complex function of the various response gains G_1, G_2, ... G_n, and depend also on the degree of correlation as well as the combining process at the output of the receiving system. However, with all contributing factors specified, we can always determine G_s and T_{op} from their basic definitions, (Appendix III).

Conclusion

The foregoing indicates that in order to evaluate $\frac{S_o}{N_o}$ of any receiving system, the system's engineer must know T_{op}, B_o, and the total input signal power S_i (having the same distribution over the various input responses as assumed in the evaluation of T_{op}).

The component designer can measure the multiple channel effective input noise temperature, T_b, (i.e., T_e, for a single response receiver) by equation (23), the gains of the various responses and the noise bandwidth, B_N. The marketer can quote these numbers. The system's engineer can use these numbers to calculate his particular system's operating noise temperature, T_{op}, by inserting them together with his signal output bandwidth, B_o, and his particular generator noise temperatures, T_g's, into the general equation (31), or any appropriate simpler form. (For instance: equation (6) for the single response receiver; equation (26) for the multiple input response receiver with signal in only one response; equation (33) for multiple input responses with equal gains and uncorrelated input signals which are arbitrarily distributed, etc.)

From this value of T_{op}, the output signal-to-noise ratio may be calculated from equation (2).

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* B_o is the bandwidth of the signal delivered to the output utilization circuit. (In the case of several coherent output signal responses appearing in different frequency bands B_O denotes the bandwidth of the signal in any one response. In the case of a superheterodyne receiving system, B_O denotes the signal bandwidth appearing in the intermediate frequency amplifier.)

**By single response receiver is meant any receiver in which only one frequency at the accessible input terminals corresponds to a single output frequency, regardless of the complexity of the gain-frequency characteristic.

† Multiple response receivers are those in which more than one frequency applied to the accessible input terminals corresponds (by way of transformations) to a single output frequency and vice versa. We denote the multiplicity of the responses by counting the number of frequencies which, if applied to the accessible input terminals of the system, con-

tribute significantly to a single output frequency within the desired output band.

[1]I. R. E. Standards 59 I. R. E. 20. S1. Proc. I. R. E. Vol. 48, pp. 60–68; Jan. 1960.

[2]J. P. Gordan and L. D. White, Noise in Maser Amplifiers - Theory and Experiment. Proc. I. R. E. Vol. 46, pp. 1588–1594; September 1958.

[3]D. O. North, "The Absolute Sensitivity of Radio Receivers", RCA Review, Vol. 6, pp. 332–344; January, 1942.

[4]H. T. Friis, "Noise Figures of Radio Receivers", Proc. I. R. E., Vol. 32, pp. 419–422; July, 1944.

[5]Standards on Receivers: Definitions of Terms, 1952. 52 I. R. E. 17. S1 Proc. I. R. E. Vol. 40, pp. 1681–1685; December, 1952.

[6]I. R. E. Standards on Electron Tubes: Definitions of Terms, 1957. 57 I. R. E. 7. S2 Proc. I. R. E., Vol. 45, pp. 983–1010; July, 1957.

APPENDIX I

I. R. E. Definition of Effective Input Noise Temperature (Reference (1), pg 68)

Definition: Effective Input Noise Temperature of a Two-Port Transducer). The input-termination noise temperature which, when the input termination is connected to a noise-free equivalent of the transducer, would result in the same output noise power as that of the actual transducer connected to a noise-free input termination.

Note 1: For heterodyne systems there is, in principle, more than one output frequency, corresponding to a single input frequency, and vice versa, an effective input noise temperature is defined for each pair of corresponding frequencies.

Note 2: The effective input noise temperature depends upon the impedance of the input termination.

APPENDIX II

I. R. E. Definition of Noise Figure [Reference (6) page 1000]

Noise Factor (Noise Figure) (of a Two-Port Transducer)

At a specified input frequency the ratio of 1) the total noise power per unit bandwidth at a corresponding output frequency available at the output Port to 2) that portion of 1) engendered at the input frequency by the input termination at the standard noise temperature (290°K).

Note 1: For heterodyne systems there will be, in principle, more than one output frequency corresponding to a single input frequency, and vice versa; for each pair of corresponding frequencies a noise factor is defined.

Note 2: The phrase "available at the output Port" may be replaced by "delivered by system into an output termination."

Note 3: To characterize a system by a noise factor is meaningful only when the input termination is specified.

Noise Factor (Noise Figure), Average (of a Two-Port Transducer).

The ratio of 1) the total noise power delivered by the transducer into its output termination when the noise temperature of its input termination is standard (290° K) at all frequencies, to 2) that portion of 1) engendered by the input termination.

Note 1: For heterodyne systems, 2) includes only that noise from the input termination which appears in the output via the principal-frequency transformation of the system, and does not include spurious contributions such as those from an image-frequency transformation.

Note 2: A quantitative relation between the Average Noise Factor $\bar{F}$ and the spot noise factor F (f) is

$$\bar{F} = \frac{\int F(f)G(f)df}{\int G(f)df}$$

where f is the input frequency, and F(f) is the ratio of 1) the signal power delivered by the transducer into its output termination, to 2) the corresponding signal power available from the input termination at the input frequency. For heterodyne systems, 1) comprises only power appearing in the output via the principal-frequency transformation of the system; for example, power via image-frequency transformation is excluded.

Note 3: To characterize a system by an Average noise factor is meaningful only when the input termination is specified.

Noise Factor (Noise Figure), Spot. See:

Noise Factor (Noise Figure) (of a Two-Port Transducer).

Note: This term is used where it is desired to emphasize that the noise factor is a point function of input frequency.

Noise Temperature (at a Port).

The temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual Port, at a specified frequency.

Note: See: Thermal Noise

Noise Temperature, Standard

The standard reference temperature T_o for noise measurements is 290°K.

Note: $kT_o/e = 0.0250$ volt, where e is the magnitude of the electronic charge and k is Boltzman's constant.

APPENDIX III

Definition

Operating Noise Temperature, T_{op}. (of a system under operating conditions).

The ratio of

1) The total noise power, N_o, delivered by the system into its output utilization circuit under operating conditions, to

2) $k\ B_o\ G_s$

where k = Boltzmann's constant

B_o = Bandwidth of signal delivered to utilization circuit (in case of several coherent output responses, the bandwidth of the signal in any one response.

$$G_s = \frac{\text{Total signal power delivered by the system into its output utilization circuit (Under operating conditions)}}{\text{Total signal power available to the system (under operating conditions) at its accessible input terminals.}}$$

Note 1: In the above definition, noise contributed by the utilization circuit to the total output noise power is assumed negligible. If significant, such contribution must be added to 1) in order to compute T_{op}.